Steven Stanek
Physics AP B

The Abbreviated Steven Summary

Chapter 17: Current and Resistance

Electric Current

Electric Current is best defined as the movement of charge over time. This means that Current (I) is determined by Change in Charge (ΔQ) divided by Change in Time (Δt). Therefore, the basic unit of change in charge is 1 Ampere (A) or 1 C/s.

In general, we say the direction of current is the positive direction although the charge is often moving in the negative direction (as charges are often electrons). However, occasionally, there are exceptions in which positive "charge carriers", moving charges, are used. (Such as Linear Accelerators)

$$I = \frac{\Delta Q}{\Delta t}$$

Current and Drift Speed

If a conductor has a cross-sectional area of "A" and if we examine a length of that conductor "Δx" then we find the volume of that area of the conductor, "AΔx". If we know the number of carriers per unit volume (the density of the carriers), "n" and the charge of each individual carrier, "q" we can multiply "q" by the number of charges per unit volume "n" by the volume "AΔx" to get the total charge in that part of the conductor.

$$\Delta Q=(nA\Delta x)q$$

Since we are often dealing with charge which is in fact moving at a speed called its drift speed, we can find the distance "Δx" by taking the drift speed "v_d" and multiplying it by the time it has been moving. This gives us the below equation.

$$\Delta Q=(nAv_d\Delta t)q$$

If we were to divide both sides of the equation by Δt we find that we have arrived at a new charge equation.

$$I = \frac{\Delta Q}{\Delta t} = nqv_d A$$

Note of drift speed 1: The charge carriers do not actually move in straight line but rather more of zig-zag path due to their collisions with atoms in the conductor. These collisions generate heat, but they continue to have a net motion in one direction at an average speed which we refer to as drift speed. Additionally, an electric field does exist in a conductor in which there is charge in motion (current) but does not exist in a static charge.

Note of drift speed 2: Drift Speeds are often very low (ex: $2.46 * 10^{-4}$ m/s). However, when a light switch is flipped or a machine turns on it seems to happen instantly. This is because electrons behave like water in a pipe. Although it may take hours for a water molecule to go from one end of the pipe to the other, the moment it is there to exert pressure, water starts coming out of the other end.

Resistance and Ohm's Law

When a voltage, "V", is applied across a conductor, the current, "I", is found to proportional to the the voltage, "V". The proportionality constant is the resistance of the conductor R, the ability of the of the conductor to decrease the current flowing through it.

$$V = IR$$

The flow of current is similar to that of water in a river of equal width and depth. Although the river has these two constants, it can still have obstructions which change the rate of flow the river. The same is true of conductors, resistance drops the flow rate of a conductor.

For many, materials, resistance remains constant over a wide ranges of voltages. This relationship is known as Ohm's law. Ohm's Law is not a law of nature and only applies to certain "ohmic" materials. Other "nonohmic" materials do not follow the simple Ohm's law relationship.

Resistivity

If a conductor has voltage applied between its ends, a charge in the middle is constantly being accelerated by the field. At the same time, the charge is colliding with atoms and thus losing speed. These collisions act somewhat like a force of friction on the charge and slow it down, thus they are origins of resistance.

In ohmic conductors, resistance is proportional to the length of the conductor, "l", (assuming a constant cross-sectional area "A"). The constant of proportionality is the constant p, or the resistivity of the material. The better the conductor, the lower the resistivity and thus the lower the resistance. Likewise, good isulators have higher resistivities and thus high resistances.

$$R = \rho \frac{l}{A}$$

Temperature Variation of Resistance

For most metals, an increase in temperture results in an increase of resistivity. This is due to the fact that the atoms in the substance are moving faster, making it more difficult for electrons to make their way through. (It is similar to trying to walk through a room of moving people, versus walking through a room full of still people). Due to this, we can find the change in p by multiplying the original p (p_0) by 1 + the coefficent of resistivity for that substance (∂) and the opposite of the change in temperature ($T - T_0$). This results in the below equation.

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

Since resistance is proportional to restivitivity through the equation $R = \rho \frac{l}{A}$, we can replace the p and p_0 in $\rho = \rho_0 [1 + \alpha(T - T_0)]$ with R and R_0. (As the l and A won't change if its the same conductor). So, we get.

$$R = R_0 [1 + \alpha(T - T_0)]$$

The carbon microphone is often used as a mouthpiece for a telephone. It consists of a steel diaphram which is in contact with a box of carbon granules. The granules are linked to one end of a circut while the diaphragm is linked to the other. When the diaphragm is hit by sound waves, it expands or contracts accordingly. These expansions and contracts increase or decrease the density of the granules and thus increase or decrease the resistivity. These increases and decreases therefore effect the resistance of the circuit and result increases or decreases in current.

Superconductors

There are some materials which below certain temperatures have virtually no resistance. These materials are known as superconductors. Superconductors exhibit the normal characteristics of standard conductors until they reach a “critical temperature” or T_c. At or below T_c their resistance drops to zero. It is so low that no voltage needs to be applied across the conductor to cause the flow of current. Steady currents have been observed to be looping in superconductors for years without decay.

Ironically, the commonly used conventional conductors: copper, gold, and silver do not exhibit superconducting abilities. We have recently discovered high temperature super conductors which act as superconductors at temperatures of 150 K or higher (vs original superconductors which had to be lower than 10K).

Some practical applications for superconductors include creating superconducting magnets, magnets with magnetic fields ten times greater than normal magnets and could therotically be used as a means for storing energy. Also, superconducting power lines could be used, this would permit highly efficient transmission of power.

Electrical Energy And Power

If there is a hypothetical circuit with a battery, resistor, and a ground (just before the current returns to the battery), we can trace the path of a test charge. The test charge, starting out at the ground would have a PE of 0. As the charge passes through the battery, it gains a PE ($PE = q\Delta V$). But as the charge passes through the resistor, it looses a good deal of PE. We can find the rate of loss of PE by putting PE over time. So: $PE / t = (\Delta Q)/(\Delta t) * V$. $(\Delta Q/\Delta t) = I$ so $PE / t = IV$. Since this is also the rate at which the charge is loosing energy (Energy / time), it must be equal to power. So: $P = \frac{PE}{\Delta t} = \frac{\Delta QV}{\Delta t} = IV$

This equation can be represented in other ways by substituting elements from $V= IR$. Here are few.

$$P = IV = I^2 R = \frac{V^2}{R}$$

Energy in Household Circuits

In houses, we use electricity for a variety of things. Ultimately we must pay for the use of the electricity. The standard unit used by power companies is the kilowatt-hour or kWh. This is actually a measure of energy, not power in that it is the amount of energy that 1000 Watts would produce in one hour. So: $1000\text{W} * 3600\text{s} = 3.6 * 10^6$ J

Chapter 18: Direct Current Circuits

Sources of EMF

The source which maintains a current in a closed is called the source of "emf" (electro-motive force). Any devices which increase the potential of circulating charges are sources of "emf." These can include batteries and generators. An emf is in essence a charge pump, something which forces charges around a circuit, the same way a water pump pumps water around a pipe. The emf is measured in volts.

In batteries, there is both a potential drop across the battery known as the *terminal voltage* and a resistance. Since the resistance is internal to the battery, the terminal voltage is NOT the EMF as a portion of the terminal voltage is lost. It is easier to split a battery into two parts, a source of EMF and a resistor in series with the source. We can find the actual EMF by looking terminal voltage and adding current x the internal resistance of the battery. SO:

$$V_{terminal} = \varepsilon - Ir \quad \text{OR} \quad V_{ter\min al} + Ir = \varepsilon$$

If the circuit has other resistors present, the emf if equal to the sum of the resistances (both internal and external) times the current.

$$\varepsilon = Ir + IR \quad \text{OR} \quad I = \frac{\varepsilon}{R + r}$$

In most cases the internal resistance of the battery is so low in comparision with other resistors, it can be disregarded.

Resistors in Series

If two resistors are connected to a source of emf in series, there is only one pathway for current to travel. This means that the same current must travel through both resistors. So, the potential drops across each one must be independent. However, the sum the potential drops across each resistor must be equal to the terminal voltage. (ex: If one resistor has a resistance of 3Ω while another has a resistance of 6Ω under a 9 V emf, the voltage across the 3Ω resistor would be 3 V while the resistance across the 6Ω resistor would be 6 V).

Since the sum of the potential drops of the resistors must equal the terminal voltage, $V_{total} = IR_1 + IR_2 = I(R_1+R_2)$ this means that the resistances can be added togethor to create a resistance which is equivalent to the individual resistors. Or, the sum of all resistors in a series circuit can be added together to create a "virtual" resistor of equivalent resistance.

$$R_{equivalent} = R_1 + R_2 + R_3 ...$$

In a series circuit, breaking a connection at any one of the resistors inhibits the flow of electricity to the rest of the circuit as all the charge must flow through the same path. This principle is used to the advantage of electrical consumers, in fuses, electrical components which automatically melt to break a circuit if too much current is flowing through it.

Resistors in Parallel

If two or more resistors are placed in a parallel circuit, each resistor gets its own charge path, meaning that the current across them is different. At the same time, the potential across each one is the same as each one is connected directly to the battery. This means the potential drop across every resistor in a parallel circuit is equal while the current is usually different (the exception being when two resistors have the same resistance). This means that total current going through a circuit can be found by adding the currents flowing through each individual resistor, or by finding the equivalent resistance by finding the inverse of each resistance, adding them together and then inverting the that. This can then be plugged into the V=IR equation to find the current flowing through a circuit.

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

Kirchhoff's Rules and Complex Circuits

Although, we can combine resistors to create a single simulated large resistor, we cannot always do this, for instance a circuit might contain two batteries, one of which is on a branch of the circuit. Under these conditions, we cannot find the equivalent resistance by combining all of the resistors.

Kirchhoff's rules are two relatively simple in the abstract but can become very confusing when we try to apply them. Kirchhoff's first rule is know as the rule of conservation of charge. Basically, it states that the current going into a junction is equal to the sum of the currents going through each of the individual branches. Kirchhoff's second rule states that the sum of the potential differences across all elements in a closed circuit is zero. In essence, the potential drop due to batteries and backflow through batteries must equal the emf in any closed loop.

Note: In a parallel circuit, there are often many loops so it is necessary to calculate each and every possible loop.

When Kirchhoff's are applied there are two rules which one should keep in mind. First, one must assign symbols and directions to all branches of a circuit. Should the direction be wrong the answers are simply the opposites of the actual answers. Second, when one applies the loop rule, one must choose the direction of the loop. The rises and falls in the loop should follow the below rules.

1. A resistor traversed in the direction of the current it results in a – change in potential.
2. If a resistor is traversed against the assigned direction of current it results in a + change in potential.
3. If a source of emf is traversed in the same direction as the current, it results in a + change in potential.
4. If a source of emf is traversed in the opposite direction of the current, it results in a - change in potential.

There are limitations on the number of times each of these rules can be used. The junction rule can be used as long as the current plugged into it is not from a previous junction rule equation. The loop rule can be used as long as a new circuit element (resistor or battery) or a new current appears in each equation.

RC Circuits

We have thus far considered only circuits with constant current. This changes in RC circuits with the addition of a capacitor. If consider a circuit with a capacitor, resistor and source of emf, we find that as time elapses the capacitor gets infinately closer to its maximum equilibrium value, but it never actually reaches its. This value, Q, can be found by multiplying the maximum voltage across the capacitor *E*, by the capacitance of the capacitor, C.

$$Q=C\mathcal{E}$$

The charge, q, which is stored in the capacitor at any given time can be found by multiplying the maximum equilibrium value of the capacitor, Q, by 1 minus the base of natural logorithms, e, raised to the -time, t, divided by the resistance of the circuit, R, times the capacitance of the capacitor, C.

$$q = Q(1 - e^{-t/RC})$$

Additionally, the voltage across the capacitor, V, can be obtained at any time by dividing the charge, q, by the capacitance of the capacitor, C.

$$V=q/C$$

Finally, the letter *T*, represents the time constant, RC. When *T*=t, e is raised to the -1 power. This means that when t=T, the value of e is inverted and then substracted from 1, making approximately 63% of the maximum equilibrium value, Q.

Discharging is very similar to charging. Like charging, the capacitor follows a parabolic curve and never fully discharges. In addition, rather than using the maximum equilibrium value, we use the initial charge, Q. Otherwise, we keep the variables t for time, R for resistance and C for current. The equation is.

$$q = Qe^{-t/RC}$$

Since the voltage across the capacitor decreases exponentially as well, we can replaces the q with the present voltage, V and the Q with the initial voltage across the capacitor, *E*.

Household Circuit (abbreviated)

Our homes contain 120V parallel circuits to power appliances. The circuits first contain a meter and a circuit breaker connected in series. The circuit breaker’s task is to open a circuit should to much current flow through the circuit and threaten to damage the wiring. Typical household circuits will go up to 15A (1800 W) although those which run high output appliances such as toasters and microwaves often go as high 30A (3600 W). Additionally, some appliances such as stoves, dryers and ovens require more power than that so an additional line of -120V volts is installed to create a potential difference of 240V. At 30A (7200 W) could be carried in one circuit.

Electrical Safety (abbreviated)

Never touch a live wire while grounded. This could result in burns, heart problems (especially if you use a pacemaker) or death (especially if you were just swimming in salty water). Some equiptment manufacturers now include a third line, a case ground in power cords which causes electricity to take the path of least resistance and go through the ground, not through your body. (Wow that sounds like an M&M commercial). Anyway, this thing is supposed to cause a circuit breaker to flip so after you’ve flipped it a few times and figured out that the its not working, you’ll get angry and stop the tainted electrical instrument to bits and pieces out of rage. Additionally, construction dudes now put these things in our kitchens and bathrooms called ground fault interrupters for the sole purpose of making us cuss and scream when we try to plug something in and the damned button keeps popping back out and making annoying noises. They are supposed to protect people but in reality are royal pain in the *&^.

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Steven's Summary

Chapter 19: Magnetism

Magnets

Note: Assume that the magnets in the following discussion are bar magnets
Iron objects are most often attracted to the ends of magnets, called poles. The poles of the magnet correspond with the earth. The south pole of a magnet would point south if the magnet was suspended from a string while the north pole would point north. These poles exert attractive or repulsive forces on each other in accordance with the age old expression: Like poles attract and opposites repel.
Unfortunately, magnetic poles cannot be isolated so regardless of how many times we cut up a magnet it has a north and a south pole. However, we can create new magnets through contact between magnets or even through induction from a nearby magnet. There are also natural magnets which are made of elements which have been naturally magnetized by Earth's magnetic field over eons. All magnets are generally broken up into groups, soft magnets which are easy to magnetise but also loose their magnetism quickly and hard magnets which are magnetize/demagnetize.
Magnets, like electrical charges create magnetic fields. Regardless of the strength of such a field, its "positive" direction is always north. These fields can be found experimentally with the use of small iron filings.

Earth's Unrealiable Magnetic Field

When we declare a magnet's North pole North, we are really declaring it North seeking as it rotates toward the North pole. Another ways of looking at this is calling Earth's North Pole South and our South Pole North. At any rate the North an South magnetic poles are similar to a gigantic magnet being buried within Earth.
As it turns out, the magnetic North Pole of the Earth is really a tiny bit north of Hudson Bay (about 1300 miles off) while the magnetic South Pole is about 1200 miles off the actual South Pole. This means that at some points on Earth's surface, a compass will line up directly with the poles while elsewhere it can be as much as 25° off. To complicate this problem it turns out that these poles are exactly fixed, but rather seem to be generated by ions in the planet's liquid interior. The strength of this field also seems to effect the rate of earth's rotation. Lastly, to complicate the problem even further, the direction of earth's magnetic field seems to have reversed several times over the past million years.

Magnetic Fields

When a charged particle is stationary, a magnetic field has no effect on it. However, when a charged particle is in motion, an electric field can effect it. The effect is zero when the particle is moving parallel to the field and it is greatest when the particle is moving perpendicular to the field. The velocity of the particle (v), the charge of the charge (q) and the strength of the field (B) also effect the force exerted on the particle (F). From this we can derive an equation for the force exerted on a particle by a magnetic field.

$$F = qvB \sin \theta \quad \text{or} \quad B = \frac{F}{qv\sin\theta}$$

In standard SI units, we can derive the value of B (using the above equation) as one “Newton per Ampere Meter” (N/(A*m)) or one Telsa (T) or one weber per square meter (1 Wb/m^2). Unfortunately, one Telsa is far to large for everyday use so we often use a gauss (G) which is one 10,000th of the strength of a Telsa. The strongest conventional magnets are around 25,000 G or 2.5 T, the strongest superconducting magnets are around 300,000 G or 30 T while Earth’s magnetic field is a measily 0.5 G.

$$1 \text{ T} = 10^4 \text{ G}$$

Please note that since the maximum strength of the force is at a 90º angle or perpendicular to the field and sin 90º = 1, qvB = the maximum force possible. Additionally, the force always runs in parallel with both v and B so if you use the tip of your middle finger as B, and your thumb as v you can use the palm of you hand as the origin of F.

Magnetic Force on a Current-Carrying Conductor

To cut through the junk which this textbook offers on this subject, like a single a charge, a wire is also affected by magnetic fields when a current flow through it. This is because current is really nothing more than a flowing charge. It also follows that if a magnetic field moves a wire in one direction turning the wire around (which makes the current flow in the opposite direction) will cause the wire to move in the opposite direction. As for the magnetic field, if it is going into the page, an X is put in the B box to symbolize the tail of an arrow if it is heading away from the page there is • to show that it is heading toward you. Otherwise the is a an arrow pointing in the correct direction.

If one was to persume that a wire a magnetic field were to cross at a 90º angle, we would know from the previous section that each charge carrier (electron) was affected by the magnetic field through the equation F = qvB (sin 90º = 1 so it was removed and v would be equal to the drift speed of the electrons). Since there are only so many electrons per unit volume , we can multiply the cross-sectional area by the length (the distance over which in charge travels in one of v_d’s time units) to get the volume. When these two are put together we get: $F = (qv_dB)(nAl)$ but qv_dnA = current so it can be replaced by current. Lastly we must consider that the wire is not always perpendicular to the magnet so we must add in a sin ϑ.

$$F_{\max} = BIl(\sin\theta)$$

The direction can be obtained by exploiting the right hand rule and placing the thumb in the same direction as the current. (Remember if the current is parallel to the magnetic field, regardless of direction, the field has no effect.)

Torque on a Current Loop

If we are to consider a rectangluar current loop placed in a magnetic field with 2 sides (the a sides) parallel to the field and the other sides, the “b” sides perpendicular, we find that the current is going in opposite directions in the two sides which are perpendicular to the field. If we then use one of the parallel sides as a frame of reference, we find that in the third plane, one of the forces caused by this intersection goes upward and the other goes downward. They would also be equally spaced from the axis of rotation, the O point. This means that we can apply a standard torque equation and find that T = F_1 (a/2) + F_2 (a/2) since F = BIl and l is equal to the b side, T= (a/2)(2)(BIb) = aBIb. Since ab is area of the rectangular loop, we find that torque is equal to the product of the magnetic field, the current and the area.

$$\tau_{\max} = BIA$$

Since this formula is not always used in situations where the magnetic field is parallel to one of the sides, we can add a sin ϑ when ϑ is the angle between the field and the plane of the loop. Additionally, the torque varies according to the number of rotations or the loop so,

$$\tau_{\max} = BIA(\sin\theta) \quad \text{OR} \quad \tau_{\max} = NBIA(\sin\theta) \text{ (when N is the \# of rotations)}$$

The Galvanometer and Its Applications

The galvanometer is a device which consists of a coil of wire which is free to rotate in the magnetic field of a permanent magnet. This coil is then connected to a spring which and a needle on a scale. This means that as current flows through the wire, it creates a force against the spring and causes the needle to move, the more current, the more the needle moves. Unfortunately, the galvanometer has several disadvantages when applied as a voltmeter or ammeter.

As an ammeter, one notices two problems. First, it has an average resistance of about 60Ω and second, a full scale deflection occurs at incredibly low currents (.001 A or less). To compensate for this, the galvonometer can be put in parallel with another resistor of a very low resistance. This allows the current voltage across the galvanometer to not effect the rest of circuit while allowing it to detect much greater amperages.

As a voltmeter a galvanometer has many of the same problems and solutions as its counterpart, the ammeter. For instance, an average galvanometer might be able to measure a maximum voltage of 0.06 V or so. Adding a resistor in series will fix this problem. Unlike an ammeter, a voltmeter must have a very high resistance.

Motion of a Charged Particle in a Magnetic Field

When a charged particle has a velocity, v perpendicular to the magnetic field B, the force is directed at a 90º angle from the particle. As the particle moves, the v changes but remains perpendicular to field (this is possible due to the third plane). This ultimately results in circular motion in which the magnetic force is always center seeking. The fact that the force is center seeking means that the magnetic force can be considered a centripetal force. So, the magnetic force, qvB is equal to the centripetal force, mv^2/r.

$$F_{\text{magnetic}} = F_{\text{centripetal}} \quad \text{OR} \quad \frac{m^2}{r} = qvB \quad \text{OR} \quad r = \frac{mv}{qB}$$

If the magnetic field was not perpendicular to the velocity, the charge would follow helical path.

Magnetic Fields of Long, Straight Wires and Ampère's Law

Through experimentation, Hans Oersted found that electric current in a wire caused all the compasses surrounding the wire to point in the direction of the circle caused by the resulting magnetic field. When the current was reversed, the field and thus the needles were also reversed. If the current was increased, the strength of the field went up. Inversly, if the distance was increased, the strength of the field went down. Lastly, he also arrived at the "Right Hand Law" that if one places his right hand on a wire with his thumb in the direction of the current, his/her fingers will curl in the direction of the magnetic field.

Using what Oersted's findings, scientists discovered that the magnetic field B, resulting from a current in a wire could be found by multiplying the current by constant µ (4π X 10^{-7} Tm /A), the permeability of free space, and then dividing by the 2π times the distance to the wire.

$$B = \frac{\mu_0 I}{2\pi r} \text{ when } \mu = (4\pi \text{ Tm/A})$$

André-Marie Ampère proposed a general way of finding the relationship between current, distance and magnetic fields. He said that the sum of all magnetic field ($B_{||}$) over given lengths, Δl were equal to the current, I, times the permeability of free space, μ_0. Through Ampère's Law, we can derive the magnetic field along a straigh wire by deciding that B is constant and that l is the entire way around or 2πr. This arrives at the B = (µI/2πr) equation.

$$\sum B_{\parallel} \Delta l = \mu_0 I$$

Magnetic Force Between Two Parallel Conductors

If there are two conductors which are parallel to eachother with current flowing the same direction, they obviously exert magnetic fields on eachother. We can find the magnitude of the magnetic field one exerts on the other through the B = (µI/2πr) equation. We can then plug this value in for B in the F = BIl equation. When the two are combined, the below equations are created.

$$F_1 = \frac{\mu_0 I_1 I_2 l}{2\pi d} \quad \text{OR} \quad \frac{F_1}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Since the two wires have currents going the same direction, they attract, otherwise, they repel each other. This rule is used to define the Ampère (the current needed for two wires, one meter apart to exert a force per unit length of 2 x 10-7 N / m.)

Induced emf and Magnetic Flux

If a battery is connected to a coil around one side of a piece of iron and a switch and on the other side, a second coil is connected to a galvanometer, Faraday found that closing the switch on the first circuit would cause the galvanometer's needle to move for a split second and then return to 0. Likewise, openning the switch would cause the needle to move the same amount in the opposite direction for a split second. Faraday said this indicated was that a changing magnetic field could produce a current in a secondary circuit, providing the secondary circuit with a source of emf.

This emf was actually a change in a quantity called magnetic flux or Φ. Magnetic flux is defined as the perpendicular component of the magnetic field ($B_\perp$) times the area over which the field is acting. This means that an increase in either the area or the field strength or making the field closer to the perpendicular will result in an increase in an increase in magnetic flux.

$$\Phi \equiv B_\perp A = BA\cos(\theta)$$

This also means that the magnetic flux is proportional to the number of field lines passing through it. If the object is perfectly perpendicular, to the field, the most line pass through it, if it perfectly parallel none do. Likewise, the bigger the area, the more lines pass through it and the stronger the field, the closer the lines and thus more pass through it.

Faraday's Law of Induction

If a magnet is moved toward a loop of wire which is connected to a galvanometer, the needle steadily moves in one direction. If the magnet is moved back the needle moves in the opposite direction. Likewise, the moving the wire toward the magnet causes the needle to move in the first direction while moving it away cause the needle to move in the second direction. This is similar to the the experiment in the above section in that there is a change in magnetic flux which is resulting in an emf in the wires. We also find that the needle further if the relative motion between the magnet and the loop is higher. This means that: The instantaneous emf in a circuit equals rate of change of the magnetic flux. Thus, if a circuit contains N, loops, the emf is equal to N, times the change in magnetic flux, $\Delta\Phi$, divided by the time interval Δt. This is called Faraday's law of magnetic induction.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The minus sign in Faraday's Law's of magnetic induction is due to Lenz's Law which states that an induced emf causes a current to maintain the original flux through the circuit. (IE: You push the magnet toward the loop and cause the current to go up, but pulling it away causes the current to go down.)

Motional emf

If we take a solid conductor (ex: a bar) and move it perpendicularly through a magnetic field, it will cause the charge to build up on one end of the bar (due to Faraday's Law of Induction). This build up will in turn cause a build up of an electrical field which will counteract the motion of charge due to the magnetic field. Eventually they will stabilize and we will be able to set the force equations for magnetic induction and electrical field equal. So, $F_e = qE$ and $F_m = qvB$ yields $qE=qvB$ or $E = vB$. From this we can find the strength of the magnetic field and thus the induced emf, by multiplying E by the length of bar, l. This potential difference will be maintained as long as the motion of the conductor and the magnetic field are maintained.

$$E = vB \quad V = E\ell = B\ell v$$

If the object in part one was turned into a closed circuit with a resistor, we see that the area is constantly increasing due to the motion of the bar while charges are being pumped through the circuit, creating a current. We can use the increasing area to find the induced emf. As the are equals the length of the bar, l times that expansion of the bar x. The magnetic field, B is constant and the Δt can be supplied by the v.

$$\varepsilon = \frac{\Delta\Phi}{\Delta t} = \frac{Bl\Delta x}{\Delta t} = Blv$$

We can use this equation to find the current, I, because we know the resistance, R.

Lenz's Law Revisited

As the bar from the previous section moves across the circuit the, the area increases. Lenz's Law states that the flux due to induced current is opposite the direction of the external magnetic flux, in this case, the flux due to the motion of the bar. Since the bar is moving and was pushed in one direction, it is absolutely vital that this is true since if it was not, the force acting on the bar would build on itself as it would induce an ever increasing flux within the bar and create energy.

Generators and Motors

If a coil of wire is connected to two rings which are connected to brushes which connect to the light bulb and it is rotated within a magnetic field, it will produce emf and thus light the bulb. This emf is steady however, since the coil is constantly rotating and its perpendicular angle is constantly changing. This means that the current produced follows a sign pattern. It peaks at 2NBlv (since there are two wires per turn of the loop) and goes down to -2NBlv and back up again. Furthermore, if the diameter of the coil is 'a' and the angular velocity, w, is known, we can find the emf by substituting (a/2)w for the v. The a * l = A and the 2's cancel, making the max emf equal to NBAw or the emf at any point equal to NBAw sin ∂. ∂ can be found by multiplying the angular velocity, w by the time, making it wt.

$$\varepsilon = 2NB\ell v\sin\theta \quad \text{OR} \quad \varepsilon = NB\ell a\omega\sin(\omega t) = NBA\,\omega\sin(\omega t)$$

In some circumstances, the two individual rings are replaced by a set of half rings which are seperated by an air space. As they rotate, they produce a direct current which gains and looses emf with time but never reverses direction.

Lastly, motors are essentially generators in reverse. Unfortunately, they have one annoying property. When a motor rotates within a magnetic field its rotation produces a "back emf", an emf which opposes the original emf (this is due to Lenz's confusing law). When the motor is first turned on or a heavy load which slows it down added, the v drops and the induced emf drops, making the motor consume more power.

Eddy Currents

If a conductor is exposed to a magnetic field while in realitive motion perpendicular to a magnetic field, the free electrons in the metal are caused to move by the motion relative to the field. The direction of the eddy currents must oppose the change that causes them so, they produce effective magnetic poles in the conductor, which repel the field and cause a retarding force. This can be used to our advantage in advanced train braking systems, but normally, the eddy currents hated and thus shielded again.

The Transformer

It turns out that we often want to change the potential of the current. We might want to step it up to carry it for long distances and then drop it to make it safe. We do this by wrapping two coils of wire around an iron square to increase the magnetic flux. The ac from the first (incoming) circuit causes a constantly varying electric field across the iron and induces an emf in the second circuit. In layman's terms, the change in V varies directly with the change in N, the number of rotations of the wire. Note, that this does not mean that power is created or destroyed as an increase in emf results in a decrease current.

$$V_2 = \frac{N_1}{N_2} V_1 \quad \text{AND} \quad I_1 V_1 = I_2 V_2$$

Steven Stanek
Physics AP B

Chapter 22: Reflection and Refraction of Light

The Nature of Light (abbreviated)

Way back when in the before time, people believed that Newton was the only god of Physics. Then Christian Huygens, a Dutch scientist unseated the king and replaced his particle theory of light with wave theory which explained the refraction and reflection of light as well as the master. The inhibitants of the scientific village shunned Huygens because he had challenged the king. Then, Thomas Young showed that the light "exhibits interference behavoir" which showed that light had to be wave since waves could cancel eachother. Then it was discovered that light was slower in liquids than air, the opposite of the particle model which would have made it higher. Finally, Maxwell put light in its place as a "high frequency electromagnetic wave" with a speed of about $3x10^8$ m/s. Newton was unseated and calm restored to the universe. BUT, hark who hath we forgotten. It was known that light could induce a charge in materials, courtesy of Hertz, but Philip Lenard showed that frequency, not intensity defined the kinetic energy of this charge. Einstein, the titan of titans came up with thet theory of light quanta which states that light is composed of "corpuscles" (whatever that means) or "discontinuous quanta of energy." Einstein said that the energy produced by the this interaction is equal to the frequency times Planck's constant, h or ($6.63 X 10^{-34}$ J/s).

$$E = hf$$

Einstein's theory retains features of both the particle and wave natures of light, the dual nature of light as electrons act as if they have been struck by photons (particle nature), but it can also be interfered with (wave nature). In essence, sometimes we think of light as a wave and other times as a particle.

Measuring the Speed of Light

There have been several attempts to measure of the speed of light. Traditional ones such as measuring the time it took light to travel between mountains or towers involve objects which are too close together to measure light accurately. The first semi-accurate measurement (ie had the correct number of demical places) was Ole Roemer's which was found using varations in the time period between eclipses on Jupiter's moon IO. Unfortunately, astronomers at the time didn't know as much about earth's orbit as they do now so his calculations were off by about a third. The next method was developed by Armand H.L. Fizeau who used the time it took light to bounce off a mirror and through a rotating disc to find the speed of light. If the disc had alot of notches, rotated at a high speed as was in a really big room, one could find the speed of light relatively accurately. This method has precision to about 5 places at its best but Fizeau was off in his second place. Using new laser methods we know know the speed of light is about: $2.997924574 X 10^8$ m/s.

Huygens' Principle

The laws of reflection and refraction follow a basic geometric principle, Huygens' Principle. This principle basically states that if we use the wavefront, the line where all points in a wave have the same amplitude and phase, as a center of propagation of the wave into wavelets, the wavelets will move outward as waves and form a new wavefront.

It is important to note that we often use ray approximation when dealing with waves, that is the waves are represented by rays, or straight perpendicular lines which are drawn in the direction light is travelling.

Reflection and Refraction

When light hits a surface it is often reflected. There are two ways in which this can happen, it can be reflected more or less perfectly in specular reflection. This is similar to light being reflected from a mirror or a puddle of water where a cohertent image can be constructed. In the second, light bounces off, but due to the roughness of a surface does not bounce back perfectly but rather more or less randomly. This form of reflection is known as diffuse reflection and is similar to light being reflected from a piece of paper. In general this book only deals with specular reflection.

One property of specular reflection is that the angle of incident or orignal ray to the normal is equal to the angle of the reflected ray to normal, when the normal is the line drawn perpendicular to the reflecting material

$$\theta_{incident} = \theta_{reflected}$$

In refraction of light, light passes through a surface but is bent in a fashion similar to the way a vehicle turns when one of its wheels touches a surface which offers more resistance than the other, it naturally turns inward toward the surface. As in reflection a normal line is drawn down the middle of the material through the intersection point between the material and the incident ray. Then in the diagonal from the incident, the refracted ray appears but at a different angle to the normal. This angle is greater if the speed of light in the material it is entering is lower, the angle decreases and if the speed of light is higher, the angle increases. This relationship is expressed in the equation. This is known as a form of Snell's Law.

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$$

The Law of Refraction

The ratio by which light speeds up of slows down after entering a new medium is known as the index of refraction or n. This index is ratio between the speed of light in a vacuum and the speed of light in a medium.

$$n = \frac{c}{v_1}$$

Because there is no real way for the frequency of the wave to be changed by entering a medium, the wavelength must change when the speed slows down ($v = f\lambda$). Thus, n can also be considered the ratio between the wavelength in vacuum and the speed of light in the substance. We can now express Snell's law as: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Dispersion and Prisms

N, the index of refraction is dependant upon wavelength. Light of different colors have different frequencies which express those colors and thus light of high frequencies (lower wavelength) bends less than light of higher wavelength (lower frequency). This means that light may enter an object known as a prism at an angle and emerge as a spectrum of light with red bent the furthest and violet going the straightest. (Order: Red, Orange, Yellow, Green, Blue, Violet)
An instrument known as a prism spectrometer is often used to study the wavelengths emitted from a light source. It works by filtering the light to make sure that it is parallel and then dispersed by a prism. This refracted and dispersed light is then viewed by the scientist through the eyepiece of telescope. The prism spectrometer can be used to identify gases which each emit light at different frequencies when hot and at low pressure.

The Rainbow

When sun shines into a rainshower, we get a rainbow. A rainbow occurs when the sunlight strikes the raindrops, passes through them but is reflected off the far side of the drop and thus exits the drop through the front. During the course of this process the light is dispersed by about 2º making the red bend further down (father from the normal) than the violent. When this is multiplied across many raindrops the result is that the individual drops are bending light and one refracted ray from each drop reaches our eye.

Huygen's Principle Applied

If you have bunch of rays of light hitting a reflective surface, each wave produces wavelets and our confusing friend Huygens said a few sections earlier. Since Huygens said that the resulting rays have to be tangent to wavelets, the reflected waves are tangent to wavelets. Since the radius of wavelets is the same when the entering and leaving the paper, the adjacent of the triangle remains the same and the hypotenuous is shared by the triangles made by the incoming and outgoing waves. In short this says that the angles are the same since 2 of 3 sides of a right triangle are the same. If this section isn't correct its because the book makes it ten times to complicated and quite frankly I cann't follow it to well (the book treats verbosity as virtue). You can also try to prove Snell's Law with the principle but I completely miss that boat.

Total Internal Reflection

When light moves from a medium with a high index of refraction to a medium of a low index of refraction, it tends to bend away from the normal. Eventually, the light is bent in such a fashion that it no longer leaving the material but rather travelling along the border of the material. This is known as "the critical angle." This angle on, light will be reflected within the medium, the greater the angle, the greater it is reflected within itself. By using Snell's Law, we can set the sin of one the outgoing ray times the substance's index of refraction equal to the second object's index of refraction time the sin of 90º (1 and therefore not needed). This results in the equation for the critical angle where the first substance is more dense than the second. Objects with a higher index of refraction therefore have lower critical angles than objects with lower indexs. $\sin\theta_{critical} = \frac{n_1}{n_2}$

Chapter 23: Mirrors and Lenses

Plane Mirrors

The plane mirror is the simplest possible mirror, and the most commonly used. In all mirrors, there is an object which is being reflected, O, and image of that object which appears on the screen, I. The distance from the actual object, O, to the mirror is p and the apparent distance from the image of the object in mirror to the mirror's surface is q or the image distance. Since a plane mirror reflects an image without altering the apparent distance of the image, p = q. Thus, the apparent height of an image, I is the same as the object's height if viewed directly. In other mirrors, the Lateral Magnification or M is defined as the ratio of the reflected image's height to the object's height. In a plane mirror M=1 so the two are always =.

$$M \equiv \frac{h_{image}}{h_{object}}$$

In a plane mirror, there is a "right-left reversal" in which something which on the right hand side in reality appears on the left hand side and things on the left hand side appear on the right. Finally, rearview mirrors use the fact that there are actually two different forms of reflection in a mirror. The first is due to light passing through the mirror and being refracted by the glass, bouncing off the silver coating and being refracted once again on its way out. The majority of light rays do this. The remaining rays are reflected by the surface of the glass, similar to being able to see yourself in a window at night. The mirror is tilted so that during the day the rays reflected from the silver coating are bounced into the driver's eyes and the one from the glass miss. During the night, the opposite is true and the rays from the back of the mirror miss while the dimmer rays from the front hit.

Real Image: Image which can be projected as it was caused by the intersection of rays

Virtual Image: Rays did not insect to create it so it cannot be projected onto a screen

Concave Spherical Mirrors

If we were cut sphere and coat the interior of the sphere with a reflective material, we get a concave spherical mirror. In this mirror, there is a center of curvature, C, a point which rests in space, and the center of the spherical segment, V, which is a point on the surface. The line drawn from V through C is known as the principle axis. If we assume that an object is also placed on this axis, we can see that rays will converge at an image point, I, and there create a real image. The distance from I to V is considered q and the horizontal distance from the object to V is considered p.

If the object is moved off the principle axis, we can presume that two rays will hit the mirror (see image 23.7). The first will originate at the object, pass through C and bounce directly back along the same path. The second bounces off V and is thus reflected at the same angle as the angle of incidence, creating a third path, the second reflected across the principle axis. The place where the 1st and 3rd rays intersect will be a distance h' below (or above) I. Since I is q from the from V, we know the horizontal component is q, and the angle is already known (the angle formed the object and the principle axis). We can thus find h' by saying that the tan of the angle is equal to h' divided by q. Since M is the ratio between h & h' and tan θ=h'/q and tanθ=h/p M = -q/p.

$$M = \frac{h'}{h} = -\frac{q}{p}$$

Through more trig we can find that the tan of the angle formed by the first incident ray to the principle axis is equal to the verticle distance of the object from the principle axis divided by p-q.

This reduces to the mirror equation: $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$

If an object, is very far away compared to q, 1/p can be considered almost zero and the rays originating from the object nearly parallel. The image point, I, can thus be considered half way between C and V and is thus declared the focal point or F. The focal length (q) is this equal to R/2. We can therefore define the focal length as the inverse of inverse of p plus the inverse of q.

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Convex Mirrors and Sign Convertions

A convex mirror is the opposite of a concave mirror in that the reflective surface is the exterior surface of a partial sphere. This means that as light hits the mirror it bounces off and is scattered in all directions, creating a small, virtual and erect image, behind the mirror when the rays would converge. The equations from the above section can be used in calculating the features of convex mirror by using sign convertions. These convertions are (taken from table 23.1):

- p is + if the object is in front of the mirror (real object)
- p is - if the object is in the back of the mirror (virtual object)
- q is + if the image is in front of the mirror (real image, behind focus on concave mirrors only)
- q is - if the image is in front of the mirrror (virtual image, convex, plane, in front of focus on concave)
- Both f and R are + if its a concave mirror
- f & R are - if its a convex mirror
- M is postive if the image is erect (all the time except if the object is in front of the focus on a concave mirror)
- M is negative if the image is invereted (in front of focus on a concave mirror only)

In order to draw a ray diagram, one needs two things, the position of the object and the center of curvature. After this, one can draw three rays, one which is parallel to the principle axis (goes thru the focus), one which is drawn thru the focal point but reflect back across the principle axis and one which goes thru the center of curvature and reflects back upon itself. Any two of these rays intersect at the location of the image. While the third acts as a check. The image produce by this construction must have the same q value as the mirror formula supplies.
NOTE: In a concave mirror, the when the object behind the focal point the image is real and inverted. However, when the object passest the focal point it the image becomes virtual and erect.

Atmospheric Refraction

The atmosphere is layer of gas of changing density and composition. For this reason, we cannot look at the atmosphere as if it has a single index of refraction but rather a constantly changing index of refraction which increases the closer one gets to the earth. This results in rays of light following a constantly bending path towards earth's surface. This results in the sun appearing to be still up when in reality the rays are being bent and thus providing a phony image of the sun. Mirages follow a similar pattern. Since in deserts and other hot environments warm air is constantly rising toward cooler, higher elevations, their is a constant change in the density of air and thus its index of refraction. The net result of this is that their are two different forms of rays which reach and observer's eyes. First, there is the one which is from the object which travels on a straight line path to the eye of the observer. Second, there is ray which initiates in the downward direction but is slowly refracted upward and bends back into the eye of the observer.

Lenses

There are two types of lenses, ones with fat centers or converging lenses and ones with fat sides and skinny centers or diverging lenses. Since these lenses are symmetrical each has two focus points, one on each side. In since it is presumed that these lenses are relatively small the focal length can be measured to the surface or to the center of the lens with relatively little loss of accuracy. Lastly, since we are presuming that the lenses in question are relatively thin, we can disregard the slight variation in the final rays produced by Snell's Law and thus presume that rays moving through the center of a lens follow a straight path (although Snell's law actually does cause very minor variations).

Using ray diagrams, we can derive the magnfication for lenses and find that it is exactly the same as that for mirrors. That is, magnification is equal to the height of the image divided by the height of original object or, it is equal to the distance from the image to the center of the lens divided by the distance of the object to the center of the lens.

$$M = \frac{h_{image}}{h_{object}} \quad \text{OR} \quad M = -\frac{p}{q}$$

Using even further derivations, we find that the equation for finding the focal length is also the same as mirrors. That is the inverse of the distance of the image to the center of the lens plus the inverse of the distance of the object to the center of the lens is equal to the inverse of the focal length. The value of f is postive in a converging lens and negative in a diverging lens (see 23.3 for more).

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

The focal length can also be found using the lens maker's equation which states the relationship between the focal length, the index of refraction, and the curvatures of its front and back surfaces.

$$\frac{1}{f} = (n-1)(\frac{1}{R_{front}} - \frac{1}{R_{back}})$$

We can located the image formed by rays in a mirror using ray diagrams, the same way we did so with mirrors. We can generate two rays, one parallel to the principle axis and passes through the focal point, the second through the center of the lens (and thus does not alter course). Where these two rays converge, one can find the image. Occassionally, a third ray, one which originates at that focal point and emerges parallel to the principle axis is drawn. In a converging lens, objects outside the focal point produce real images while objects within the focal point produce virtual images. Lastly, lenses can be combined by using one's output as the other's input to find the results of multiple lens combinations. The final or overall magnification is known as the product of the magnifications of the lenses.

Lens Aberrations

Most of what we have seen in this chapter presumes there are certain qualities of lenses and mirrors which are given in order for the methods provided to work perfectly. Unfortunately, there are minor fluxuations in lenses and mirrrors which are known as aberrations. Some are due to refraction (the reason the there is the “thin” in the thin lens equation) which means that their is not a single focal point as the rays do not collide in a specific point. Another form known as spherical abberration is due to the fact that light rays hitting the edges of lenses and mirrors miss the focal point. In mirrors this can be corrected by using expensive parabolic mirrors. Lastly, there is chromatic aberration which is due to the fact that different wavelengths of light disperse (refract at slightly different angles), this results in each color having a slightly different focal point. This can be corrected by combining converging and diverging lenses in which one has a higher index of refraction than the other. (The different indices of refraction are needed because otherwise the lenses might cancel eachother out, I think).

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Physics AP B

Chapter 24: Wave Optics

Conditions for Interference

Light waves can interfere with eachother in the same manner as sound or travelling waves. Unfortunately, light waves travel very quickly and have very small wavelengths, thus there are a series of conditions for noticable interference of light waves to occur: they must be in the same phase (follow the same pattern), they must have the same wavelength and the superposition principle must apply. Unfortunately, the first condition (which can be achieved by sound waves by simply connecting to speakers to the same source) cannot be achieved in lights because lights undergo random changes every 10^{-8} s, therefore light sources are noncoherent. One thing we can do however, is to use a single light source and run the light through two small to seperate the original beam. Any change occurring to the light in one slot will also occur in the light in the second slot.

Young's Double-Slit Interference

If one feeds light through a single slit and then through a double slit, two small slits very close together, and projects it onto a screen. One can actually see the wave interference pattern in which there are various degrees of light and dark where the center is brightest and the brightness drops as one proceeds away from the center. The areas of brightness are due to the fact that there, the two light waves constructively interfere thus magnifying their intensity, this occurs when both waves are travelling the same distance or one has to travel one whole wavelength more than the other. Conversely, if a wave has to travel 1/2 wavelength more than the other, the two waves will be at opposite points and engage in destructive interference which cause a dark spot. If we define the distance between the slots to be d from the slots to the screen as L, and the midpoint between the slots as Q, the distance the light from slot 1 has to travel as r_1 and the distance the light from slot 2 has to travel as r_2, we find that by knowing the angle between the line from the point on viewing screen where the rays collide to the Q and a line originating at Q and running perpendicular to slots and the screen this angle, θ, can be related to the difference in the paths in that the sin of θ times d is equal to the difference between r_1 and r_2.

$$\delta = r_2 - r_1 = d\sin\theta \qquad \text{when } \delta = \text{path difference (dif between } r_1 \text{ \& } r_2\text{)}$$

We find that this path difference can be found if for the brightest slot if one ray is an integral number of wavelengths longer than the other. So, if we consider M to be the number of the bright spot. We can find the path difference by multiplying m by the wavelength. Likewise, if we wish to find a dark spot, we can do that by adding 1/2 to m. Additionally, since the number for the wavelength is very tiny, the number for d is tiny and the number for L relatively large, we find that for the first few value of m, θ practically equals 0. Therefore, we can set y equal to L times the wavelength and m over d. (m is shifted 1/2 for dark spots)

$$\delta_{bright} = m\lambda \qquad \delta_{dark} = (m - \tfrac{1}{2})\lambda \qquad y_{bright} = \frac{\lambda L m}{d} \qquad y_{bright} = \frac{\lambda L(m + \tfrac{1}{2})}{d}$$

Change of Phase due to Reflection

In addition to Young's two slit method, one can also place a light source above a mirror which is perpendicular to a screen. The light from the source hits the mirror, bounces up and meets the light which leaves the source on the screen. However, the positions of the light and the dark points are reversed (opposite of Young's) because when the waves reflect they under go 180º degree phase shift during reflection. This shift is due to the same reasons that the transverse waves in strings are reflected back inversly when they reach a rigid surface, however if they a more easily moveable object such as a ring, there is no phase change in the reflected wave. Likewise when light hits a more dense medium there is a 180º phase change but when it hits a less dense medium there is no phase change.

Interference in Thin Films

The multiple colors observed in thin films such as soap bubbles and oil are due to interference. This interference results from the fact that one of the rays of light bounces off the surface and is reflected, causing a 180º phase shift while the second passes through the film and reflects off the bottom which doesn't result in a change of the phase but does require the ray to go father. Thus, distance travelled by the wave which goes through the film is two times the thickness of the film while the ray which bounces off the surface undergoes a 180º (1/2) phase shift. We can find the equation for constructive interference by compensating for the phase shift by adding the 1/2.

$$2t = (m + \tfrac{1}{2})\lambda_n$$

Since the wavelength of light in material is equal to in wavelength in a vacuum divided by the index of refraction, we can substitute $\frac{\lambda}{n}$ for λ_n.

$$2t = (m + \tfrac{1}{2})(\frac{\lambda}{n}) \qquad \text{which is the same as} \qquad 2tn = (m + \tfrac{1}{2})\lambda$$

Likewise, we can find the equation for destructive interference by simply removing the 1/2.

$$2tn = (m + \tfrac{1}{2})\lambda$$

Another method known as Netwons rings places a converging lens on top of a mirror, making one ray reflect within the lens while the other goes through and reflects off the mirror. Thus, the ray which reflects off the mirror has a 180º phase shift while the other does not but has to travel a greater distance. Thus the equations for this are approximately the same as those for the thin films.

Diffraction

If a beam of light actually travelled in straight lines as proposed by the geometric model of light, then all the light leaving a single slit would be going in a straight line and Young's double slit interference would not work. However, light tends to go in all directions when it goes through a slit not in a straight line. This property, known as diffaction is what allows light to go around corners, through holes and other obsticals.

One example of diffication, known as the fresnel bright spot, is the fact that if a penny is placed in front of a light source, a bright spot will appear at the center of the penny's shadow because that is where the central maximum should be. (This would not work if one used the geometric model) A second form of diffraction known as Fraunhofer diffraction presumes that the light rays being sent to a screen are approximately parallel. If this is so, the geometrical model would say that they would create a laser like effect on the screen. However, if the screen is positioned far enough away or a converging lens introduced, one observes a bright fringe with alternating dark and bright bands.

Single Slit Diffraction

We have previous assumed that slit are point sources of light, however, it turns out that light from one portion of that slit can interfere with another portion of light from the same slit. If we divide the slit into two part such as in figure 24.15, we find that the ray originating at the bottom of slit must travel a longer distance than the ray originating at the bottom of the slit. This difference it the paths, the path difference, is equal to the size of the slit, a, divided by two because it is the distance from the center to the side, times the sine of the angle formed by the rays to the a line perpendicular to the slit's surface. We can thus come up with the equation:

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad \text{OR} \quad \sin\theta = \frac{\lambda}{2}$$

However interference does not have to only occur between waves of that are from an end and the center of the slit. It could occur between points which are a fourth of slit length apart, a sixth, an eight or any other even number. Thus, we can multiply the equation above by a non-zero integer constant m, to indicate the proportion of the slit which is interfering with the other parts. (ex: if m =1 it is half a slit length, m =2 is a fourth, m = 3 is a sixth, etc)

$$\sin\theta = m\frac{\lambda}{a}$$

The above equation indicates where dark fringes are formed. In general the bright fringes are halfway between them with a central bright fring with twice the width of the other fringes in the center.

Polarization

If we look at an electromagnetic wave, it is a wave, moving in one direction with light in all the planes surrounding it. If we limit light to a single plane, we have created polarization. The first and most common way to polarize light is through polarizing materials such as polaroid which absorb light going in all but one plane in which it transmits the waves. Since the light waves are based upon vector addition, if we put light through one polarizing filter and we can rotate a second filter or analyzer form a point where no light goes through it to a point where the same amount of the light going through the first filter is going through the analyzer. We can find the intensity of light leaving the system of two or more polarizing filters by multiplying the initial intensity of the light by the square of the cosine of the angle between them (the angle is 0º when they are oriented in the same manner and 90º when they the orientations are opposite.

$$I = I_0\cos^2\theta \text{ (known as Malus's law)}$$

In addition to being able to polarize by selective absorption, we can polarize through reflection at certain angles. If light strikes a surface, it has two components, one parallel to the surface and one perpendicular. The parallel one reflects reflects better than the perpendicular one so the beam is partly polarized. The light which is refracted is also polarized. Thus, when a merry point is reached at which the all the light is reflected across the parallel and thus all polarized. This angle is known as the polarizing angle or Brewster angle and is always due to an incident ray which is a right angle from the refracted ray. Through some trig, it is found to be the tangent of n.

$$n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\sin\theta_2} = \frac{\sin\theta_p}{\sin(90-\theta_p)} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$

In a final form of polarization, light is absorbed by a gas or a liquid and then rereleased in a different direction. This is known as polarization by scattering.

Chapter 26: Einstein's Special Theory of Relativity

Intro

It turns out that Newton's laws don't really apply at speeds close to the speed of light. For instance a particle travelling at 99% of the speed of light (c) through a potential difference, should be able to travel at nearly 2 twice the speed of light by doubling the potential difference. It turns out that this is not true, no matter how hard we try, we cannot make the particle pass the speed of light. Additionally, Einstein's set of laws uses encompasses Newton's. Einstein's law rely on the following. A) The laws of physics are the same in all inertial reference systems. B) The speed of light in a vacuum ($3X10^8$ m/s) is independent of the motion of the observer.

The Principle of Relativity

In essence the principle of relativity says that speed is relative to the frame of reference one is in. For example, if you sit down in your chair, you think of yourself as stopped, that is not in motion relative to the earth. However, relative to the sun you are in motion in at the speed earth is presently orbitting it at and you are moving at a different speed relative to other celestial bodies. Thus, one must use a frame of reference in which you define what motion in a system is relative to. This is known as an inertial frame of reference and there is no preferred frame of reference for Newton's Laws of mechanics.

The Speed of Light

If the frame of motion didn't matter, would it not also make sense the frame not matter for electricity or magnetism. But this brings up a very troubling prospect, that is that if these were true, the speed of light would not be constnat. That is if you were moving relative to another tree, and you shined a flashlight at that tree, the light you emitted would be going at c + your velocity. Thus, you would have given light a higher relative speed. On the other hand, if you were to say that light, like other forms of waves had to travel in specific medium known as "ether" to scientists, you would have imposed a frame of reference on the motion. In addition, all attempts to detect this medium had failled.

Disproving the Ether Theory

In 1887, a pair of scientists attempted to find an interference pattern generated by the relative change in the speed of light due to earth's motion neither they or anyone else who ever repeated the experiment could. Some guy by the name of Al Einstein figured out why.

Einstein's Principle of Relativity

In short, the principle of relativity states that the laws of physics are the same in all inertial frames. Thus, the speed of light will always be the same regardless of where it is measured as relative motion is unimportant. This means that all that unimportant stuff like time and distance is really relative anyway.

Consequences of Special Relativity

One of the major consequences of Special Relativity is that time is relative. That is if we have a light bulb in relative motion to earth, on a rocket ship and light reflects off a mirror which is attached to rocket ships, an observer on the ship considers himself stopped and therefore thinks that the light only had to go to the mirror and come back again (i.e. twice the distance to the mirror). However, an observer on earth would see that the both the ship and the mirror are in relative motion to him and thus, the light would have to travel on a diagonal to the mirror and bounce back at an angle in order for the guy on the ship to see it. It is clear that the observer on earth would think that the light had to go farther. Thus, if the person in the rocket ship were to calculate the time it took the light to reach him, he would use v = d/t so t = d/v and plug 2d in for d and c in for v and find that his time was 2d/c. The guy on earth would use the pythagorean thereom to find the value for his t is equal to 2d over the square root of c^2 - v_{rocket}^2. We can factor out the c^2 from this equation and find a final equation in terms of the t on the rocket. That is:

$$\Delta t = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c^2\sqrt{1 - {}^{v^2}\!/_{c^2}}} = \frac{\Delta t'}{\sqrt{1 - v^2 - c^2}} = \gamma \Delta t' \quad \text{when} \quad \gamma = \frac{1}{\sqrt{1 - {}^{v^2}\!/_{c^2}}}$$

The time interval known as Δt’ in the above example is the time on it apparently takes on the rocket ship, or the proper time. The change in time due the fact that the earth was in a different frame of motion is known as time dilation.

It turns out that length also apparently changes due to relative motion. That is if we measure the distance between two points from a source which is relatively at rest, this distance will be longer than the distance measured from a source which is moving relative to them. Thus, the proper length, the length measured from a source relatively at rest, is equal to the length due to relative motion times the square root of 1- v^2/c^2.

$$L = L'\sqrt{1 - {}^{v^2}\!/_{c^2}} = \frac{L'}{\gamma}$$

Relativistic Motion

In conventional mechanics, it is said that p=mv. Unfortunately, if one looks at this system from the perspective of relativity one finds that this is not entirely true as momentum must be conserved in all reference systems. However, since much of what goes on the real world is much smaller than the speed of the light, this difference is so tiny that it can often be ignored. Nevertheless, the equation for finding the momentum at higher speeds it is the original mv divided by γ .

$$p = \frac{p_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 v$$

Mass and the Ultimate Speed

We have already seen that length and time change according to relative motion well, Al Einstein said so does mass. Mass at any given speed can be found by multiplying the mass at rest by γ .

$$m = \frac{m_0}{\sqrt{1 - {}^{v^2}\!/_{c^2}}} = m_0\gamma$$

This equation means that as the speed of an object approaches the speed of light, it approaches infinate mass. Since to accelerate an object of infinate mass, you would require infinate energy, you cannot make any material travel at the speed of light. Additionally, one can find the relative momentum of an object by multiplying its relative mass by its velocity.

Relativistic Addition of Velocities

One would think that if a spaceship was moving forward at .9c and then fired a rocket in the forward direction at .7c faster, the rocket would exceed the speed of light which is in direct contradiction with the postulate that no object can go faster than the speed of light. Einstein solved this dilema by comparing the velocities of the objects as seen in the frames (u and u') to the velocity between the frames, v.

$$u = \frac{v + u'}{1 + \frac{vu'}{c^2}}$$

Relativisitic Energy

Einstein found that the equation for relativistic kinetic energy is $mc^2 - m_0c^2$. Additionally, he found that matter could be converted into energy through his famous mass-energy equivalence equation.

$$E = mc^2 = KE + m_0c^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}$$

Occassionally we like to represent energy in terms of momentum, rather than speed. This is done by combining the equations $E=mc^2$ and p=mv.

$$E^2 = p^2c^2 + (m_0c^2)^2$$

If we are dealing with a particle of no mass (such as a photon) we can greatly simplify this equation to:

$$E=pc$$

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Physics AP B

Chapter 27: Quantum Physics

Blackbody Radiation and Planck's Hypothesis

All objects emit a form of radiation, known as thermal radiation. We witness this all the time. At low temperatures it is in the infared portion of the electromagnetic spectrum while at higher temperatures, it can go all the way through the visible spectrum to ultra-violet. (We see red coals or white light from burning magnesium as signs of this radiation) The relationship arrived at is knwon as Wien's displacement. It is relateds the wavelength at the highest point in the curve to the temperature emitted.

$$\lambda_{\max} T = 0.2898 \times 10^{-2} m \cdot K$$

Conventional reasoning regarding how and why thermal radiation is emitted indicates states that it is due to emissions from accelerated charged particles near an object's surface. This definition provided totally inaccurate when referring to black bodies or devices such as a hollow sphere with a single hole in it, which absorb almost all energy incident on them. In such a body, as the temperature increased, the total amount of energy emitted also increased, but it failed to follow the pattern outlined by conventational physics. The solution to this was known as Planck's hypothesis which basically states that matter contains "submicroscopic oscillators" or "resonators" which were spread throughout the surface of the cavity of a black body. Each individual black body could only give off a discrete amount of radiation which was based upon an integer called the quantum number (n), the frequency of resonance (f), and a constant known as Planck's constant (h).

$$E_n = nhf \qquad h = 6.626 \times 10^{-34} J \cdot s$$

These "resonators" emit discrete units of light known as quanta or photons, by jumping from energy state to state. Only by changing energy states can quanta absorb or radiate energy. Since most jumps are single jumps, we can set n to 1 and find the difference between two energy levels depends solely upon the frequency, f.

$$E=hf$$

The Photoelectric Effect

If light is incident on a metal, it can cause a current. This is known as the photoelectric effect and those electrons which are emitted are known as photoelectrons. If we set up an experiment in which there is a power source with a variable voltage in an incomplete circuit with two plates of metal across from eachother and light incident on one of them, we get a current after the voltage passes a certain point, known as the stopping potential. Once above that point, the current is can be increases to a certain extent with the help of an increase in potential, but primarily by an increase in light intensity. However, light intensity has no effect on the kinetic energy of the photoelectrons nor can it allow for current below the stopping potential, regardless of how high this is. The kinetic energy only depends upon the charge of an electron and the voltage of circuit.

$$KE_{\max} = eV_0$$

Einstein explained this effect more throughly in 1905. Help proposed that.

1. No electrons are emitted below a certain cutoff frequency
2. As the light frequency exceeds the cutoff, the photoelectric effect is observed. The number of photoelectrons emitted is proportional to light intensity but the KE of the photoelectrons is not related to the light intensity.
3. The maximum KE of the photoelectrons increases with light frequency.
4. Electrons are emitted from the surface almost instaneously (contrary to classical physics)

Einstein also universalized Planck's quantum energy equation, by saying it could provide the energy of the photons in the light beam based upon their frequency. Lastly, he considered light to have more of a particle nature than a wave nature in that it was composed of photons each of which had the stated energy level. This meant that when the photons hit a metal, they transferred their energy to the electrons in the metal, thus making the energy in the electrons also equal to hf. However, since the electrons had to pass through a metal surface, they had to overcome this barrier, by forfeiting a certain portion of their energy, ϕ.

$$E_{\text{photon}} = hf \qquad KE_{\max} = hf - \phi$$

Since Einstein found a way of calculating the KE, one can presume that if hf < ϕ, the KE is less than 0 and thus, the cutoff point has been passed. The wavelength of the cutoff can be through the below. Any wavelength greater than the result will be cutoff.

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\phi/h} = \frac{hc}{\phi}$$

X-Rays

X-Rays are waves of wavelength of around .1 nm and frequencies higher than ultraviolet. They have the ability to penetrate many solids with relative ease (and are thus used for determining bone structure, etc). One device which gives off X-Rays is the X-Ray tube, a device which acts something like a lightbulb for X-Rays. If one plots the frequency of an X-Ray emitted from such a tube against the intensity of the X-Rays, one finds that it is generally a continous spectrum which spike at certain places. These spikes are only observed after the accelerating voltage exceeds a certain point, known as the threshold voltage. It is found that X-Ray radiation is due to the emittion of radiation by electrons during deceleration (acceleration in a negative direction). An electron is deflected and accelerated by an atom when it nears the positively charge nucleus. Classical physics states that a particle must radiate energy when it is accelerated, so photons must be released which in turn make the electron loose KE. Should it loose all of its energy in a single collision we could find the electrons initial energy (eV), by using the E=hf equation.

$$eV = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

Thus, we can alter this equation to find the shortest-wavelength of radiation produced.

$$\lambda_{\min} = \frac{hc}{eV}$$

Note: Radiation does not have a particular wavelength is because it is due to multiple electron collisions, not a single one.

Diffraction of X-Rays by Crystals

Crystals are mostly ionic solids which means that they have an alternating pattern of atoms at a fixed distance from eachother. We can use such solids to determine the wavelengths of X-Rays as they are too high frequency to use a classical diffraction gratting. If we fire X-Rays at such a substance, we find that the X-Rays are incident on these sheets at a certain angle. Some X-Rays will bounce off the top surface while others will go another level down and bounce off the second layer. We can find the path difference between the two by taking the sin of the angle of incidence and multiplying it by twice the distance between the atoms (once for incoming and once for outgoing). If this path difference is equal to an integer multiple of the wavelength there is constructive interference.

$$2d\sin\theta = m\lambda$$

Compton Scattering

Compton found that scattering X-Rays caused their wavelengths to increase, indicating that their energies were considerably lower. He explained this by stating that the photons behave the same way as conventional particles in collisions, indicating that both momentum and energy must be conserved. Thus one can relatively easily find the shift in wavelength due to scattering by multiplying Planck's Constant by 1 minus the cosine of the angle of scattering and dividing the rest mass times the speed of light.

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_0 c}(1 - \cos\theta)$$

Pair Production and Annihilation

Since a photon has energy and good ol' Al Einstein said $E=mc^2$, we can turn a photon into matter. Unfortunately, since a photon has no charge, we cannot create an electron, so through a process by the name of Pair Production, a really high energy photon can simulateously create an electron and a positron (an antimatter particle of the same mass but a charge opposite that of the electron). We can find the frequency of the electron required to produce this by using $E=mc^2$.

$$hf_{\min} = 2m_0c^2$$

Through an alternative process known as pair annihilation, a positron and an electron can combine to create two photons, each of which travels at a given speed in an opposite direction (necessary to conserve momentum).

Photons and Electromagnetic Waves

Light clearly has a wave nature as is demonstrated by the interference experiments discussed in wave optics. It also has a particle nature as described in this section. Since this contradiction exists, we cannot describe light in terms of a single classical definition and must instead accept that light has dual nature, that is it depends on the specific phenomenon being observed. In the frequencies of visible light, the energy of each individual electron is so low that so many must be transmitted that light appears to be essentially a wave. At higher frequencies, the light exhibits characteristics of both waves and particles and in the gamma ray area, it is almost entirely particle and roundabout methods are required to locate its wave side.

The Wave Properties of Particles

Normally, we try to classify stuff as either particles or waves. As light shows, its not that easy, indeed, French physicist Louis de Broglie postuled that matter exhibits wave characteristics. He said that since we know from relativity that the momentum of a photon is equal to its energy divided by the speed of light and the energy can be found by E = hf (E=hc/λ), we find that the momentum of a photon can be found by combining these equations, or we can find the wavelength based on momentum.

$$p = \frac{E}{c} = \frac{hc}{c\lambda} = \frac{h}{\lambda} \qquad \text{OR} \qquad \lambda = \frac{h}{p} = \frac{h}{mv}$$

De Broglie also postulated that matter obeys the E=hf relationship. These seem to indicate that mass and velocity, two matter related quantities can be equated with the wavelength and frequency which are wave concepts. This was supported by evidence collected in 1927 through experiments into the interference of electrons with eachother.

The Wave Function

Erwin Schrodinger proposed a “wave equation” which deals with the change in waves through space and time. Although this is a key element in quantum mechanics, it is too difficult for high school or beginning college students to derive or even intelligently understand. However, it is based upon the wave function, a quantity which differs per particle and depends on both position and time. It also allows for the dual nature of light. For instance, in a wave the wavefunction value, Ψ, is based upon E^2 while in matter, it is based upon the amplitude. The Ψ function can be used to describe a single particle while Ψ^2 can be used to find its location.

The Uncertainty Principle

Classical mechanics states that we can measure anything to unlimitted accuracy, we can always make a measurement device which is more accurate. However, Heisenburg’s uncertainity principle indicates that the precision of measuring an electron’s position is limitted to the change in the x value times the change in momentum which is greater than or equal to Planck’s constant. This is because the measuring procedure itself interferes with the end results. This equation can also be represented in terms of energy and time interval.

$$\Delta x \Delta p \geq \frac{h}{4\pi} \qquad \text{OR} \qquad \Delta E \Delta t \geq \frac{h}{4\pi}$$

Chapter 28: Atomic Physics

Early Models of the Atom

You already know about the early models of the atom devised by Rutherford etc, with the electrons orbitting the nucleus. This model has several distinct problems: It does not explain why atoms emit certain discrete frequencies of radiation and it puts electrons in centripetal acceleration, a problem which would eventually lead to the collision of the electrons and the nucleus.

Atomic Spectra

If we fill a glass tube with gas, run a current through it and analyze the output through a device called a spectroscope, we find that each different element emits its own unique set of frequencies. This set of frequencies is called the element's emission's spectrum. For hydrogen, one can find the wavelength (and therefore the frequency through the below equation.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{4} - \frac{1}{n^2}\right) \quad \text{(n is an integer greater than 2; } R_H \text{ is } 1.097 \text{ X } 10^7 \text{ m}^{-1}\text{)}$$

In addition to emitting light at these frequencies, each element will also absorb light at the same frequencies. That is, if we put the gas in the tube and expose it to radiation of all frequencies, the same frequencies which appear when the current is run through the tube are absent while the rest appear.

The Bohr Theory of Hydrogen - The Stuff that makes Rocket Science look easy

Neils Bohr figured out why certain gases have certain absorbtion and emition frequencies by combining a bunch on classical physics and a bunch of new fangled physics. The derivations involve the combinations of equations and are long and annoying. Basically, Bohr started out with 4 principles.

1. The electrons move in circular orbits about the nucleus due to Coulomb attraction.
2. Only certain electron orbits are stable. When in these orbits, electrons don't emit radiation.
3. Radiation is emitted by electrons when they jump from one orbit to another. The frequency of the radiation emitted in the jump is related to the change in the atom's energy but independent of the frequency of the electron's orbital motion. (E_i - E_f = hf)
4. The "size" of electron orbits are those for which the electron;s orbital angular momentum about the nucleus is and integer multiple of h/2π ($\hbar$).

Through some complicated derivations, we arrive at the following equations.

$$E = -\frac{ke^2}{2r} \qquad E_n = -\frac{13.6}{n^2}eV \qquad \frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Modifications of the Bohr Theory

The Bohr theory only applies to hydrogen-like atoms (one electron atoms). These atoms can be used with the Bohr theory by substituting Ze^2 for e^2. In order to make this theory more general, Arnold Summerfield introduced the orbital quantum number, l, which has a value between 0 and the principle quantum number, n, this gave rise to the s,p,d,f,g and h subshells. The maximum number of electrons allowed in any such subshell is 2(2l+1). Additionally, in order to account for changes in the spectrol emission when exposed to strong magnetic fields, the orbital magnetic quantum number was introduced, m_l (which ranges from l to -l). Lastly, more spectrometers revealed additional information which indicated that spectral lines are actually two different closely spaced lines (the splitting is known as fine structure). This required yet another quantum number: the spin magnetic quantum number or m_s.

De Broglie and the Hydrogen Atom

In his hydrogen atom theory, Bohr postulated that angular momentum was dependant on $\hbar$ ($h/2\pi$) times an integer, n. Unfortunately, no one could figure out why only an integer. Why couldn't an electron orbit at $\hbar$*1.5? De Broglie connected Bohr's hydrogen theory with his own wave characteristics of materials theory and found that an electron orbit which was divisible by an electron's wavelength would work. In essence, an orbit with circumference λ, 2λ, 3λ, etc would work, but nothing in between. Thus $2\pi r = n\lambda$! Through this, one could simply derive Bohr's original equation.

Derivation: $2\pi r = n\lambda$ & $\lambda = \frac{h}{mv}$ so $2\pi r = \frac{nh}{mv}$, $mvr = n\hbar$

Know: $\lambda = \frac{h}{mv}$

Quantum Mechanics and the Hydrogen Atom

The allowed energies in Quantum mechanics depend only upon the principle quantum number, n (this is because the energies of allowed states must correspond to the Bohr theory). The values for l (orbital quantum number) & m_l (magnetic quantum number) depend on math, not wierd scientific assumptions are thus set in stone. They are:

- The values for n can range from 1 to infintity in integer steps
- The values for l can range from 0 to n-1 in integer steps
- The values for m_l can range from -l to l in integer steps

The Spin Magnetic Quantum Number

It turns out that electrons spin, in somewhat the same fashion as the earth spins on its axis. Thus, a fourth quantum number, the spin magnetic quantum number, is introduced. This number depends on the direction of spin if a electron is spinning counterclock wise, it is known as spin up and $m_s = +.5$. If the electron is spinning clockwise, it is known as spin down and $m_s = -.5$.

Electron Clouds

Basically, the probability of finding an electron at any given location is dependant on the square of the wave function (which is dependant on n, l, an m_l). Thus, if we look at the graph of where an electron might be, we get a steep hill, with the peak being the most probable location. Thus, we can never say where an electron is but we can say where it is highly probable that the electron is. It is important to note that the locations predicted by Bohr are the most probable.

The Exculsion Principle (NOT the Periodic Table)

The state of an electron in an atom is specified by the four quantum numbers: n, l, ml, and ms. These apply to electrons in all atoms, not just hydrogen. Additionally, Pauli's exclusion principle states that in each individual atom, no two electrons share the same set of these four quantum numbers. As a general rule, the lowest energy subshells are filled first (remember that a subshell contains 2(2l+1) electrons). When a subshell or shell is filled and there is large gap between energy levels, a noble gas is formed.

Atomic Transitions

When light of all frequencies is incident on an atom, only photons with an energy equal to that of the energy of the excited state minus the energy of the ground state will be able to cause electrons to jump to higher state. All other photons are disregarded. Likewise, spontaneous emission occurs once an electron has jumped; it emits a photon with the same energy as the difference in the energy stimulated and ground energy states. Lastly, there is stimulated emission which occurs when an electron is at the stimulated state and is hit by light of the same energy as the difference between the ground and stimulated states. This causes the emission of a second photon of equal energy and forces the electron back to the ground state. (Remember for this stuff: E=hf in photons)

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Physics AP B

Physics Book VI

Chapter 29: Nuclear Physics

Some Properties of Nuclei

In describing atoms, we sometimes use: the atomic number, Z, the neutron number, N, and the mass number, A. (One can find the neutron number by subtracting the atomic number from the mass number.) We generally represent the nuclei of atoms as ${}^{A}_{Z}X$ when X is the atomic symbol. The Z value is often removed since it can be found by looking up the atomic symbol on a periodic table.

All nuclei of a given element have the same number of protons but may have a different number of neutrons. Forms of an element with different numbers of neutrons are known as isotopes (they have the same Z but different N and A values)

The proton is 1836 times as massive as the electron and has approximately the same mass as a neutron. The proton's mass is considered equal to one unified mass unit (atomic mass unit), thus the atomic mass of an isotope is equal to the sum of the numbers of protons and neutrons. Additionally, mass is occassionally refered to in terms of rest energy, provided by good old, $E=m_0c^2$.

Rutherford tried to measure the size of a nuclei by trying to find the Coulomb repulsion force between it and the incoming alpha particle (Helium nucleus) and equating this with the KE of the alpha particle. He found it to be about 2×10^{-14} m. Later experiments found a more general equation which relies on the constant $r_0 = 1.2 \times 10^{-15}$ m and the mass number of the atom.

$$r = r_0 A^{1/3}$$

Note: The radius of atoms is so small it is commonly measured in femtometers (fm) which are equal to 10^{-15}m.

Since a nucleus contains so many positive charges, it requires a force to hold it together. This force is known as nuclear force. It is an attractive force between the neutrons and protons and quite strong at very short distances. However, as the atomic number and thus the number of protons increases, the Coulomb repulsion between the protons increases and the number of neutrons exceeds the 1:1 ratio common for smaller atoms to compensate. Once the atomic number hits 83, even the addition of neutrons cannot compensate for the Coulomb force and the nucleus becomes unstable.

Binding Energy

The mass of a nucleus is always slightly less than the sum of all the nucleons. This "loss of mass" is due to the fact that the energy of the binding is equal to the mass difference times c^2. If one looks at the bonding energy to atomic mass graph, the bonding energy is highest at A = 60. This element is the most stable.

Radioactivity

The spontaneous emittion of radiation by atoms is known as radioactivity. There are three types of radiation emitted by radioactive substances: alpha particles or helium nuclei, beta particles which are either electrons or positrons and gamma rays which are high frequency electromagnetic waves. We can distinguish between these particles by exposing a beam consisting of all three to a perpendicular magnetic field (one pointing into the page). In accordance with the right hand rules, the alpha particles will go up, the beta particles down and the gamma rays straight. Lastly, the alpha particles are the least powerful while gamma rays are the most powerful.

As time passes radioactive stuff emits radiation and slowly becomes less radioactive. This is known as decay. The number of nuclei which decay in any given second can be found by multiplying the number of nuclei, N, by a decay constant λ, by the change in time, Δt. Likewise we can find the decay rate (decays per unit time), R, by dividing ΔN by Δt or by multiplying λ by N. However, the most general equation for decay gives us the number of nuclei present at any given point in time based upon the original number of nuclei, N, decay constant, λ, and time t.

$$\Delta N = -\lambda N \Delta t \qquad R = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N \qquad N = N_0 e^{-\lambda t}$$

Additionally, one can also express radioactive decay in terms of half life, that is the time it takes half the particles of a substance to decay. $T_{1/2}$ or half life is expressed in terms of decays per second or becquerels or 3.7 X 10^{10} decays per second known as Curies.

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The Decay Processes

There are different decay processes for alpha, beta and gamma particles. For alpha particles a radioactive elements such as ^{238}U is emit an alpha particle, meaning that two neutrons and two protons are removed from the nucleus. This loss of particles changing the atomic number, of the parent, dropping it by two and inturn changing the type of element it is. This also decreases the atomic mass by 4 amu as four nucleons have been removed. This process of changing one element into another is known as trasmutation. Also note that in transmutation, some of the matter from the original parent is transformed into kinetic energy and most of which goes into the alpha particle as momentum is conserved in the collision. The KE can be found by using the below equation.

$$KE = \frac{p^2}{2m} \text{ (ex: } {}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He\text{)}$$

In the process of beta decay, one of the neutrons in the nucleous of an atom becomes a proton and an electron is emitted. This process increases the atom's atomic number by one but leaves the atomic mass nearly the same. The small change in atomic mass also produces energy but unlike alpha particles, the bulk of this energy does not go into the beta particle but rather into a third type of particle known as a neutrino which is chargeless, practically massless and interacts poorly with mattter. If an electron is emitted as a beta particle, it is an antineutrino is emitted if a positron is emitted, a neutrino is emitted. (The symbol for neutrinos is v and antineutrinos are $\bar{v}$)

$$\text{(ex: } {}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{v}\text{)}$$

Gamma rays are emitted for many of the same reasons are lower frequency electromagnetic waves, they are due to protons and neutrons jumping energy levels within the nucleus. Since the protons are neutrons are more massive than the electrons their jumps release more energy. In general these jumps are the result of an excite state brought on by alpha or beta decay so an equation for gamma decay is generally two parts.

(ex: $\begin{array}{l} {}^{12}_{5}B \rightarrow {}^{12}_{6}C^{*} + {}^{0}_{-1}e \\ {}^{12}_{6}C^{*} \rightarrow {}^{12}_{6}C + \gamma \end{array}$) (NOTE: *represents excited state)

Natural Radioactivity

There are two kinds of radioactive nuclei, the naturally radioactive ones, those found in nature and the artifically radioactive ones which are produced in labs. In naturally radioactive ones, the decay series starts out with an element with a really long half lives which decrease with each decay until a stable end product is reached. This allows long lasting elements to continuously replenish elements with shorter half lives.

Nuclear Reactions

We can solve nuclear reaction much like equations. For example: if we have the process ${}^{4}_{2}He + {}^{14}_{7}N \rightarrow X + {}^{1}_{1}H$, we can add the atomic masses of He and N and substract the atomic mass of H, getting (4+14-1 or 17). We can then add the atomic numbers of He & N and substract the atomic number of, H getting 8. Thus, the value for X is ${}^{17}_{8}O$.

In reactions, as previously mentioned, the sum of the masses on the opposite sides of an equation are usually not equal. This is due to a change in KE, due to the collisions. This often means that the mass of the reactants is greater than that of products and missing mass has been transformed into energy which can be found through e=mc^2. This energy is known as the q value.

Unfortunately, things are not always this simple. Sometimes, an equation ends with more mass than it began with courtesy of the KE of one of the initial particles being turned into mass. Unfortunately, since both momentum and KE must be conserved, the energy cannot be found through a straight e=mc2 conversion but rather requires the below relationship where m is the mass of the particle and M is the mass of the target.

$$KE_{\min} = \left(1 + \frac{m}{M}\right)|Q|$$

Chapter 30: Nuclear Energy and Elementary Particles

Nuclear Fission

Nuclear Fission occurs when a really big nucleus such as Uranium is broken into two smaller daughter atoms and some free neutrons as well as alot of KE. This is stimulated through the collision of a neutron and the aforementioned atom. This temporaily creates a new atom (such as ^{235}U being hit by a neutrons and turning into ^{236}U). However this intermediate atoms is very unstable to decays quickly as the Coulomb force over powers the strong force, the result is two new atoms and 2-3 free protons. By finding the difference in binding energy per nucleon between the parent and the daughter nuclei and multiplying it by the number of nucleons, one can find the amount of energy released.

For ^{235}U: ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow X + Y + neutrons$

How To Make A Nuclear Reactor -- For Dummies

So, you wanta make a nuclear reactor huh? Well, its pretty easy. You want to start a chain reaction which is self sustaining but you don't want more than one neutron from each atom to hit another otherwise it'll blow up like an A-Bomb. You also don't want less than one neutron from each atom to another atom because the reaction will slow down. No siree, you want one neutron from each atom to atom and blow up its nucleous causing a good'ol healthy nuclear chain reaction. You do this by regulating the number of neutrons by using cadinium control rods to absorb'um. Also, there's this other wierd problem in that neutrons come outa those Uranium nuclei pretted dang fast, so fast that they are likely to make ^{238}U absorb them but not ^{235}U. Ol' Encirco Fermi figured this one out. You stick carbon atoms (or today heavy water) in with the reaction and those neutrons bump into the atoms and slow down so you got yourself a nice little nuclear reaction which is heating water which is vaporizing, going through a pipe and heating more water which vaporizes but helps condense the first water. The new steam now turns a turbine and guess what you've got? You guessed it power!!!!

Oh a tad bit about nuclear safety. The powerplants cann't quite blow up like bombs and radiation is heavily contained. As for the threat of a meltdown, this "China Syndrome" thing (based on the incorrect assumption that a reactor meltdown would go all the way through the Earth to China) thats a real threat as the radioactive material would just kinda dig a hole and contanimate our ground water "Oops." Also, this nuclear fuel stuff is just radioactive junk so you cann't make an A-Bomb outa it or anything (but you can make dirty nukes!!!)

Nuclear Fusion

The same way we can pull one big piece apart and make two tiny pieces + energy, we can put a bunch of little hydrogens together and get alot of energy. This is through the process of fusion. All modern nuclear weapons use fusion as it is much more destructive. All stars also use fusion as the source of heat, temperature, emittions, etc. They do this through the following three step process.

Step one: Two hydogens combine into deutrium (a hydrogen isotope): ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e + v$

Step two: The deutrium from 1 combines with hydrogen to produce helium: ${}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2} + \gamma$

Step three: One of following happens

- The Helium combines with hydrogen producing more helium: ${}^{1}_{1}H + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{0}_{1}e + v$
- Two Heliums combine to form Helium and Hydrogen: ${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + {}^{1}_{1}H + {}^{1}_{1}H$

Unfortunately, the sun and other stars are massive and can sustain these gigantic fusion reactions for a long time at relatively low temperatures. We humble humans cannot do so on Earth. So, we've got to figure out another way. Although we can blow up H-Bombs (fusion bombs) we cannot create self sustaining thermonuclear reactors. One of the main problems in fusion is that the temperature of fuel must be high enough to overcome Coulomb repulsion. In order to do this, the state of hydrogen must be increased to that of plasma. In addition to that, we must beef up the plasma ion density n and the plasma confinement time, t in such a way that the exceed approximate constants. Unfortunately, the problem of plasma confinement has yet to be solved as it cannot be confined by physical stuff so magnetic fields and other stuff are needed.

$nt \geqq 10^{14} s/cm^{3}$ For a Deutrium-tritium interaction
$nt \geqq 10^{16} s/cm^{3}$ For a Deutrium-deutrium combination

For the obvious reasons expressed above, a deutrium-tritium combination is the best. All reactions on earth are different from those in stars. They are expressed below.

Deutrium-deutrium #1: ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$ Q=3.27 MeV
Deutrium-deutrium #2: ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H$ Q=4.03 MeV
Tritium-deutrium #1: ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$ Q=17.59 MeV

Elementary Particles

After the discovery of the neutron, we thought we had found all the particles. We were wrong, experiments found hundreds of new tiny particles with incredibly short half lives. So far 300 have been found. This seemed to indicate that the universe was indeed chaotic as there seemed to be no rhyme or reason to the particles. Finally, it was established that all the particles with the exception of electrons, photons and a few other are made of one kind of particle known as a quark. These quarks are placed in tightly bound patterns to create other particles.

The Fundamental Forces in Nature

It turns out that there are four fundamental forces in nature: electromagnetic, gravity, strong and weak.

- Strong: Strong is most powerful of the forces and hold the nucleus together (short ranged)
- Weak: Weak is a weaker force which distorts nuclei and causes Beta decay (short ranged)
- Electromagnetic: EM is 10^{-2} times the strength of strong
- Gravitational: The force of gravity is 10^{-38} times the strength of strong

All three of these times of forces are quantized with particles: Strong uses gluons, weak uses bosons, EM photons and gravity uses gravitions (which have not been discovered and *may* not exist)